Preface

Early examples of topological concepts in physics*

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In the mid-1940s, S. S. Chern published an "intrinsic proof" of a generalization of the Gauss–Bonnet Theorem to 4-dimensions. The paper led to the Chern Class and Chern Numbers, to the new exciting field of global differential geometry, and to new important topological concepts in other areas of mathematics. Andrei Weil was one mathematician who was greatly impressed. He wrote an enthusiastic review of the paper which became very influential.

A few years later, in 1946–1949, several totally unexpected new elementary particles were discovered by experimental physicists. They were of different kinds, with very different quantum numbers, and quickly became physicist's center of attention.

One day in 1948, I was present at a lunch in which Weil told Fermi his speculation that these new particles might be related to some topological classification ideas in geometry. Neither Fermi, nor I, nor others at that lunch, understood what Weil had meant that day by his speculation across the boundary of math–physics.

Many years later, in the mid-1970s, after I learned from Jim Simon elements of fiber bundle geometry and related concepts, I realized Weil maybe speculating that day about possible relationships between the new particles plus their new quantum number with topological concepts such as the Chern Numbers. For details please see Ref. 1.

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In an article published in 2012,[2] I discussed in some detail the following early entry of topology into physics:

- The Aharonov–Bohm experiment proposed theoretically in 1959, and verified experimentally by Tonomura in 1983–1986.
- In the early 1950s physicists used the new computers to calculate the vibrational frequency distribution of crystals, and were surprised to find unexplained ups

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and downs in the spectra. Were they real? Or just quirks of the computation? The puzzle was resolved in a 1953 paper of Van Hove which introduced topology, viz. Morse Theory, into physics.

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That topological concepts are important in physics is now well known, especially in phenomena/problems involving Abelian or non-Abelian phases. Here is an example which shows that in one problem in *classical* Maxwell theory, topology already plays an essential role:

Consider

An EM field interacting with both an electric charge e and a magnetic charge g,

a problem which had been considered by Dirac in 1931.[3] The electromagnetic potential (i.e. the connection), when analytically continued, forms a complicated ***nontrivial*** manifold. The action integral $\boldsymbol{a}$ is then definable ***only modulo*** $\boldsymbol{4\pi eg}$.[4]

If one tries to quantize this theory, à la Feynman's path integral, one would be dealing with the quantity

$$\exp(i\boldsymbol{a}/\boldsymbol{\hbar}),$$

which is meaningful only if

$$2eg/\boldsymbol{\hbar} = \text{an integer}.$$

This condition, first given by Dirac, is thus a *consequence* of the topology of classical Maxwell theory.

References

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3. P. A. M. Dirac, *Proc. R. Soc. London A* **133**, 60 (1931).
4. T. T. Wu and C. N. Yang, *Phys. Rev. D* **14**, 437 (1976).